

NMSSM+

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Abstract

It is well known that the scale invariant NMSSM has lower fine-tuning than the MSSM, but suffers from the domain wall problem. We propose a new improved scale invariant version of the NMSSM, called the NMSSM+, which introduces extra matter in order to reduce even more the fine-tuning of the NMSSM. The NMSSM+ also provides a resolution of the domain wall problem of the NMSSM due to a discrete R -symmetry, which also stabilises the proton. The extra matter descends from an E_6 gauge group and fills out three complete 27-dimensional representations at the TeV scale, as in the E_6 SSM. However the $U(1)_N$ gauge group of the E_6 SSM is broken at a high energy scale leading to reduced fine-tuning. The extra matter of the NMSSM+ includes charge $\pm 1/3$ colour triplet D-fermions which may be naturally heavier than the weak scale because they receive their mass from singlet field vacuum expectation values other than the one responsible for the weak scale effective μ parameter.

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1 Introduction

The recent discovery of a candidate Higgs-like boson with a mass around $\sim 125\text{--}126$ GeV [1, 2] is tremendously exciting since it may provide a window into new physics Beyond the Standard Model (BSM), for example Supersymmetry (SUSY) [3, 4, 5, 6, 7].

In the Minimal Supersymmetric Standard Model (MSSM) the lightest Higgs boson is lighter than about 130–135 GeV, depending on top squark parameters (see e.g. [8] and references therein). A 125 GeV SM-like Higgs boson is consistent with the MSSM in the decoupling limit. In the limit of decoupling the light Higgs mass is given by

$$m_h^2 \approx M_Z^2 \cos^2 2\beta + \Delta m_h^2, \quad (1)$$

where Δm_h^2 is dominated by loops of heavy top quarks and top squarks and $\tan \beta$ is the ratio of the vacuum expectation values (VEVs) of the two Higgs doublets introduced in the MSSM Higgs sector. At large $\tan \beta$ we require $\Delta m_h \approx 85$ GeV, which means that a very substantial loop contribution, nearly as large as the tree-level mass, is needed to raise the Higgs boson mass to 125 GeV, leading to some degree of fine-tuning.

In the light of such fine-tuning considerations, it has been known for some time, even after the LEP limit on the Higgs boson mass of 114 GeV, that the fine-tuning of the MSSM could be ameliorated in the scale invariant Next-to-Minimal Supersymmetric Standard Model (NMSSM) [9]. With a 125 GeV Higgs boson, this conclusion is greatly strengthened and the NMSSM appears to be a much more natural alternative. In the NMSSM, the spectrum of the MSSM is extended by one singlet superfield [10, 11, 12, 13] (for reviews see [14, 15]). In the NMSSM the supersymmetric Higgs mass parameter μ is promoted to a gauge-singlet superfield, S , with a coupling to the Higgs doublets, $\lambda S H_d H_u$, that is perturbative up to unified scales. The maximum mass of the lightest Higgs boson is

$$m_h^2 \approx M_Z^2 \cos^2 2\beta + \frac{\lambda^2 v^2}{2} \sin^2 2\beta + \Delta m_h^2, \quad (2)$$

where here $v = 246$ GeV. For $\lambda v > M_Z$, the tree-level contributions to m_h are maximized for moderate values of $\tan \beta$ rather than by large values of $\tan \beta$ as in the MSSM. For example, taking $\lambda = 0.6$ and $\tan \beta = 2$, these tree-level contributions raise the Higgs boson mass to about 100 GeV, and $\Delta m_h \sim 75$ GeV is required to achieve a Higgs mass of 125 GeV. This is to be compared to the MSSM requirement $\Delta m_h \gtrsim 85$ GeV. The difference between these two values (numerically about 10 GeV) is significant since Δm_h depends logarithmically on the stop masses as well as receiving an important contribution from stop mixing.

In the NMSSM $\lambda \sim 0.6$ is the largest value in order not to spoil the validity of perturbation theory up to the GUT scale for $\tan \beta \sim 2$ and for $k \sim 0.2$ (where λ and k are evaluated at M_Z) [16]. However it has been known for some time [17] that the presence of additional extra matter allows larger values of λ to be achieved. For example, adding three families of $5 + \bar{5}$ extra matter at a mass scale of 300 GeV increases the largest value to $\lambda \sim 0.7$, for the same parameters as before [16]. For example, taking $\lambda \sim 0.7$ and $\tan \beta \sim 2$, these tree-level contributions raise the Higgs boson mass to about 112 GeV, and $\Delta m_h \sim 55$ GeV is required to achieve a Higgs mass of 125 GeV. This is to be compared to $\Delta m_h \sim 75$ GeV required for $\lambda = 0.6$. The difference between these two values (numerically about 20 GeV) is more than the difference between the MSSM and

the NMSSM, and can lead to a further significant reduction in fine-tuning. The above discussion shows that there is an argument from fine-tuning for extending the NMSSM to include extra matter.

An example of a model with extra matter is the Exceptional Supersymmetric Standard Model (E₆SSM) [18, 19, 20, 21, 22, 23, 24, 25, 26], where the spectrum of the MSSM is extended to fill out three complete 27-dimensional representations of the gauge group E₆ which is broken at the unification scale down to the SM gauge group plus an additional gauged $U(1)_N$ symmetry at low energies under which right-handed neutrinos are neutral, allowing them to get large masses. The three 27_{*i*}-plet families (labelled by $i = 1, 2, 3$) contain the usual quarks and leptons plus the following extra states: SM-singlet fields, S_i ; up- and down-type Higgs doublets, H_{ui} and H_{di} ; and charged $\pm 1/3$ coloured, exotics D_i and \bar{D}_i . The extra matter ensures anomaly cancellation, however the model also contains two extra SU(2) doublets, H' and \bar{H}' , which are required for gauge coupling unification [20]. To evade rapid proton decay a \mathbb{Z}_2 symmetry, either \mathbb{Z}_2^{qq} or \mathbb{Z}_2^{lq} , is introduced and to evade large flavour changing neutral currents an approximate \mathbb{Z}_2^H symmetry is introduced which ensures that only the third family of Higgs doublets H_{u3} and H_{d3} couple to fermions and get vacuum expectation values (VEVs). Similarly only the third family singlet S_3 gets a VEV, $\langle S_3 \rangle = s/\sqrt{2}$, which is responsible for the effective μ term and D-fermion masses. The first and second families of Higgs doublets and SM-singlets, which do not get VEVs, are called “inert”. The maximum mass of the lightest SM-like Higgs boson in the E₆SSM is [18]

$$m_h^2 \approx M_Z^2 \cos^2 2\beta + \frac{\lambda^2 v^2}{2} \sin^2 2\beta + \frac{M_Z^2}{4} \left(1 + \frac{1}{4} \cos 2\beta\right)^2 + \Delta m_h^2, \quad (3)$$

where the extra contribution relative to the NMSSM value in equation (2) is due to the $U(1)_N$ D -term. The Higgs mass can be larger due to two separate reasons; firstly the value of λ may be larger due to the extra matter and secondly there is a $U(1)_N$ D -term contribution equal to $\frac{1}{2}M_Z$ ($\frac{3}{8}M_Z$) GeV for low (high) $\tan \beta$. For example for $\tan \beta \sim 2$, and $\lambda \sim 0.7$, the E₆SSM can completely account for a Higgs boson mass of 125 GeV at the tree-level, without the need for any extra contribution from radiative corrections, i.e. with $\Delta m_h \sim 0$.

Although the D -term from the low energy $U(1)_N$ gauge group appears to help with fine-tuning, by increasing the tree-level Higgs mass, in fact it leads to a new fine-tuning problem associated with the non-observation of the Z'_N gauge boson. The reason is that the singlet VEV s , which is responsible for both the effective μ term and the Z'_N mass, must be quite large, in practice $s > 5$ TeV for $M_{Z'_N} > 2$ TeV, which is the current experimental limit [27]. As we shall see, such a large singlet VEV is unnatural if that singlet is responsible for the effective μ term, due to singlet D -terms entering the Higgs potential. Furthermore, a large singlet VEV implies a large effective μ term, at least for non-small λ , which also leads to fine-tuning. It might appear that if λ is small then the effective μ term may be made small so that we are back to the fine-tuning situation of the MSSM. However, as we shall see, the tree-level potential involves the Z'_N mass squared explicitly and can only be balanced by the effective μ -squared term, leading to large and unavoidable tree-level fine-tuning.

In this paper we propose a model called the NMSSM+, containing the extra matter content of the E₆SSM, but without a low energy $U(1)_N$ gauge group, this being broken close to the unification scale by a high energy mechanism which does not give rise to

mass for any of the components of the 27_i states above. The absence of the $U(1)_N$ gauge group immediately removes all related D -terms from the low energy theory, and we return to a situation similar to the NMSSM with extra matter where fine-tuning may be lowered as discussed above, without encountering any new fine-tuning problems due to the large singlet VEV. In order to achieve the smallest fine-tuning possible, we want to lower the singlet VEV s as much as possible, while maintaining a large value of λ in order to increase the tree-level Higgs mass, so that the effective μ term is as small as possible. If only the third generation SM-singlet acquires a VEV, $\langle S_3 \rangle = s_3/\sqrt{2} = s/\sqrt{2}$, then this implies that the coloured exotics D_i , \bar{D}_i should also have low masses of order μ , and their non-observation may imply a lower limit on μ , leading again to increased fine-tuning. In order to avoid this, unlike in the E_6 SSM we shall allow the first and second family SM-singlets also to acquire VEVs $\langle S_\alpha \rangle = s_\alpha/\sqrt{2}$. To minimise the fine-tuning associated with the μ parameter we assume $s_\alpha \gg s$ with s being responsible for the smaller effective μ term, and s_α being responsible for the larger masses of the D_i , \bar{D}_i exotics. Another difference from the E_6 SSM is that we shall generate cubic singlet interactions S_i^3 as in the NMSSM (for all three singlets $i = 1, 2, 3$), which breaks the Peccei-Quinn (PQ) symmetry. The associated domain wall problem which arises when the accompanying discrete \mathbb{Z}_3 symmetry is broken will be avoided in our model, however. This is due to the fact that in the NMSSM+ instead of \mathbb{Z}_2^{qq} or \mathbb{Z}_2^{lq} we impose either \mathbb{Z}_4^{qq} or \mathbb{Z}_4^{lq} . These are R -symmetries under which the superpotential itself transforms. Note that, although all three families of singlets acquire VEVs, only the third generation of Higgs doublets acquire VEVs $\langle H_{d3}^0 \rangle = v_d/\sqrt{2}$ and $\langle H_{u3}^0 \rangle = v_u/\sqrt{2}$, as in the E_6 SSM.

The layout of the remainder of the paper is as follows: In Section 2 we briefly review the NMSSM and E_6 SSM and give an overview of the NMSSM+. This treatment may be sufficient for those readers only interested in the phenomenology of the NMSSM+. In Section 3 we summarise the superpotential and symmetries of the high energy NMSSM+, which is similar to the E_6 SSM but contains a $U(1)_N$ breaking sector and somewhat different symmetries, and show how it leads to the low energy NMSSM+ outlined in Section 2. In Section 4 we discuss the important aspects of the NMSSM+ and explain the reason for and implications of the various symmetries in detail. The tree-level fine-tuning problem of the E_6 SSM is explained in Section 5 and the lower expected fine-tuning in the NMSSM+ is explained. The concluding section is Section 6.

2 Comparing the NMSSM, E_6 SSM, and NMSSM+

2.1 Overview of the NMSSM

The renormalisable, scale invariant superpotential of the NMSSM [10, 11] is

$$W^{\text{NMSSM}} = \lambda \hat{S} \hat{H}_d \hat{H}_u + \frac{k}{3} \hat{S}^3 + W_{\text{Yukawa}}, \quad (4)$$

where W_{Yukawa} is the usual MSSM-like Yukawa superpotential terms involving Higgs doublets

$$W_{\text{Yukawa}} = h_{ij}^N \hat{H}_u \hat{L}_{Li} \hat{N}_j^c + h_{ij}^U \hat{H}_u \hat{Q}_{Li} \hat{u}_{Rj}^c + h_{ij}^D \hat{H}_d \hat{Q}_{Li} \hat{d}_{Rj}^c + h_{ij}^E \hat{H}_d \hat{L}_{Li} \hat{e}_{Rj}^c. \quad (5)$$

The gauge symmetry is that of the SM and the superfield \hat{S} is a complete gauge singlet. When the scalar component of \hat{S} acquires a VEV this VEV generates an effective μ term, coupling the Higgs doublets $\hat{H}_{(d,u)}$. Alternative non-scale invariant models known as the Minimal Non-minimal Supersymmetric SM (MNSSM), the new minimally-extended supersymmetric SM, the nearly-Minimal Supersymmetric SM (nMSSM), and the generalised NMSSM have been considered elsewhere [12]. By contrast in this paper the extension of the NMSSM we consider, namely the NMSSM+, will be scale invariant to very good approximation.

2.2 Overview of the E_6 SSM

We first recall that the E_6 SSM [18, 19, 20, 21, 22, 23, 24, 25, 26] may be derived from an E_6 GUT group broken via the following symmetry breaking chain:

$$\begin{aligned} E_6 &\rightarrow SO(10) \otimes U(1)_\psi \\ &\rightarrow SU(5) \otimes U(1)_\chi \otimes U(1)_\psi \\ &\rightarrow SU(3) \otimes SU(2) \otimes U(1)_Y \times U(1)_\chi \otimes U(1)_\psi \\ &\rightarrow SU(3) \otimes SU(2) \otimes U(1)_Y \otimes U(1)_N. \end{aligned} \quad (6)$$

In practice it is assumed that the above symmetry breaking chain occurs at a single GUT scale M_X in one step, due to some unspecified symmetry breaking sector,

$$E_6 \rightarrow SU(3) \otimes SU(2) \otimes U(1)_Y \otimes U(1)_N, \quad (7)$$

where

$$U(1)_N = \cos(\vartheta)U(1)_\chi + \sin(\vartheta)U(1)_\psi \quad (8)$$

and $\tan(\vartheta) = \sqrt{15}$ such that the right-handed neutrinos that appear in the model are completely chargeless. The $U(1)_N$ gauge group remains unbroken down to the TeV energy scale. Three complete 27 representations of E_6 then also must survive down to the TeV scale in order to ensure anomaly cancellation. These 27_i decompose under the $SU(5) \otimes U(1)_N$ subgroup as follows:

$$27_i \rightarrow (10, 1)_i + (\bar{5}, 2)_i + (\bar{5}, -3)_i + (5, -2)_i + (1, 5)_i + (1, 0)_i, \quad (9)$$

where the $U(1)_N$ charges must be GUT normalised by a factor of $1/\sqrt{40}$. The first two terms contain the usual quarks and leptons, whereas the final term, which is a singlet under the entire low energy gauge group, contains the (CP conjugated) right-handed neutrinos N_i^c . The second-to-last term, which is charged only under $U(1)_N$, contains the SM-singlet fields S_i . The remaining terms $(\bar{5}, -3)_i$ and $(5, -2)_i$ contain three families of up- and down-type Higgs doublets, H_{ui} and H_{di} , and charged $\pm 1/3$ coloured exotics, D_i and \bar{D}_i . These are all superfields written with hats below.

The low energy gauge invariant superpotential can be written

$$W^{\text{E}_6\text{SSM}} = W_0 + W_{1,2}, \quad (10)$$

where $W_{0,1,2}$ are given by

$$W_0 = \lambda_{ijk} \hat{S}_i \hat{H}_{dj} \hat{H}_{uk} + \kappa_{ijk} \hat{S}_i \hat{\bar{D}}_j \hat{D}_k + h_{ijk}^N \hat{N}_i^c \hat{H}_{uj} \hat{L}_{Lk} \\ + h_{ijk}^U \hat{H}_{ui} \hat{Q}_{Lj} \hat{u}_{Rk}^c + h_{ijk}^D \hat{H}_{di} \hat{Q}_{Lj} \hat{d}_{Rk}^c + h_{ijk}^E \hat{H}_{di} \hat{L}_{Lj} \hat{e}_{Rk}^c, \quad (11)$$

$$W_1 = g_{ijk}^Q \hat{D}_i \hat{Q}_{Lj} \hat{Q}_{Lk} + g_{ijk}^q \hat{\bar{D}}_i \hat{d}_{Rj}^c \hat{u}_{Rk}^c, \quad (12)$$

$$W_2 = g_{ijk}^N \hat{N}_i^c \hat{D}_j \hat{d}_{Rk}^c + g_{ijk}^E \hat{D}_i \hat{u}_{Rj}^c \hat{e}_{Rk}^c + g_{ijk}^D \hat{\bar{D}}_i \hat{Q}_{Lj} \hat{L}_{Lk}, \quad (13)$$

with $W_{1,2}$ referring to either W_1 or W_2 .

If one neglects the E_6 violating bilinear terms $\hat{H}_{ui} \hat{L}_{Lj}$ and $\hat{D}_i \hat{d}_{Rj}^c$ then one can see that at the renormalisable level the gauge invariance ensures matter parity and hence LSP stability. All lepton and quark superfields are defined to be odd under matter parity \mathbb{Z}_2^M , while \hat{H}_{ui} , \hat{H}_{di} , \hat{D}_i , $\hat{\bar{D}}_i$, and \hat{S}_i are even. This means that the fermions associated with \hat{D}_i , $\hat{\bar{D}}_i$ are SUSY particles analogous to the Higgsinos, while their scalar components may be thought of as colour-triplet (and electroweak singlet) Higgses, making complete 5 and $\bar{5}$ representations without the usual doublet-triplet splitting.

In order for baryon and lepton number to also be conserved, preventing rapid proton decay mediated by \hat{D}_i , $\hat{\bar{D}}_i$, one imposes either \mathbb{Z}_2^{qq} or \mathbb{Z}_2^{lq} . Under the former the lepton, including the RH neutrino, superfields are odd and under the latter both the lepton and the \hat{D}_i , $\hat{\bar{D}}_i$ superfields are odd. Under the former W_2 is forbidden and under the latter W_1 is forbidden. Baryon and lepton number are then both conserved at the renormalisable level, with the \hat{D}_i , $\hat{\bar{D}}_i$ interpreted as diquarks in the former case and leptoquarks in the latter case.

A further approximate flavour symmetry \mathbb{Z}_2^H is also assumed. It is this approximate symmetry that distinguishes the third (by definition, “active”) generation of Higgs doublets and SM-singlets from the second and first (“inert”) generations. Under this approximate symmetry all superfields other than the active $\hat{S} = \hat{S}_3$, $\hat{H}_d = \hat{H}_{d3}$, and $\hat{H}_u = \hat{H}_{u3}$ are odd. The inert fields then have small couplings to matter and do not radiatively acquire VEVs or lead to large flavour changing neutral currents. The active fields can have large couplings to matter and radiative electroweak symmetry breaking (EWSB) occurs with these fields. In particular the multi-TeV scale VEV $\langle S \rangle = \langle S_3 \rangle = s/\sqrt{2}$ is responsible for breaking the $U(1)_N$ gauge group and generating the effective μ term and D-fermion masses. In particular we must have $s > 5$ TeV in order to satisfy $M_{Z'_N} > 2$ TeV, which is the current experimental limit [27], leading to large fine-tuning.

2.3 Overview of the NMSSM+

At high energies, just below the GUT scale, the NMSSM+ theory includes the matter content and gauge group of the E_6 SSM, including the $U(1)_N$ gauge group. In order to break the $U(1)_N$ gauge group we introduce a renormalisable term $\hat{\Sigma}(l\hat{\mathcal{S}}\hat{\bar{\mathcal{S}}} - M_\Sigma^2)$, where the two extra SM-singlet superfields $\hat{\mathcal{S}}$ and $\hat{\bar{\mathcal{S}}}$ have $U(1)_N$ charges Q_S^N and $-Q_S^N$ respectively (where Q_S^N is also the $U(1)_N$ charge of the usual E_6 SSM SM-singlets S_i) and $\hat{\Sigma}$ is a complete $G_{\text{SM}} \otimes U(1)_N$ singlet superfield with l being a dimensionless Yukawa coupling

constant. This superpotential breaks the $U(1)_N$ at the intermediate scale M_Σ *. Since \hat{S} has the same gauge charges as \hat{S}_i , we propose that the R -symmetry of the theory is used to forbid superpotential terms such as $\hat{S}\hat{H}_d\hat{H}_u$ and $\hat{\Sigma}\hat{S}\hat{S}$ (where \hat{S} takes the place of \hat{S}_i or vice versa) so that the hierarchy between the Σ scale and the EWSB scale can be naturally maintained. In addition, various non-renormalisable terms are also included, all controlled by extra symmetries as discussed in the next section. In particular, some of the non-renormalisable terms yield low energy cubic singlet couplings of the form $\hat{S}_i\hat{S}_j\hat{S}_k$, including the NMSSM cubic singlet coupling.

At low energies, the scale invariant NMSSM+ contains the matter and Higgs content of three 27 dimensional superfield representations of E_6 , minus the three RH neutrinos \hat{N}_i which being complete singlets may get very large masses, leaving the three quark and lepton families, \hat{Q}_{Li} , \hat{u}_{Ri} , \hat{d}_{Ri} , \hat{L}_{Li} , \hat{e}_{Ri} ; three families of Higgs doublets, \hat{H}_{di} and \hat{H}_{ui} ; three families of colour triplet and antitriplet states, \hat{D}_i and $\hat{\bar{D}}_i$; and three SM-singlets, \hat{S}_i , where we define $\hat{S} = \hat{S}_3$ and $\hat{S}_\alpha = \hat{S}_{1,2}$, and similarly for the Higgs doublets. The low energy superpotential of the NMSSM+ obeying all of the symmetries of the model is approximately that of the NMSSM *plus* an extra sector,

$$W^{\text{NMSSM}+} \approx W^{\text{NMSSM}} + W^{\text{extra}}, \quad (14)$$

where W^{NMSSM} is the same as the NMSSM superpotential in equation (4), while W^{extra} includes the extra terms associated with the couplings of the extra two families of Higgs doublets and singlets and three families of colour triplets,

$$W^{\text{extra}} \approx \lambda_{\alpha\beta\gamma}\hat{S}_\alpha\hat{H}_{d\beta}\hat{H}_{u\gamma} + \kappa_{\alpha ij}\hat{S}_\alpha\hat{D}_i\hat{D}_j + \frac{k_{\alpha\beta\gamma}}{3}\hat{S}_\alpha\hat{S}_\beta\hat{S}_\gamma + W_{1,2}, \quad (15)$$

where $i, j, k \in \{1, 2, 3\}$ whereas $\alpha, \beta, \gamma \in \{1, 2\}$. We have neglected couplings (other than those in $W_{1,2}$) that are suppressed under an approximate \mathbb{Z}_3^{HD} symmetry. $\hat{S} = \hat{S}_3$ is responsible for the effective Higgs μ term of the active third generation Higgs doublets $\hat{H}_{(d,u)} = \hat{H}_{(d,u)3}$ ($\mu = \lambda\langle S \rangle$), while $\hat{S}_{1,2}$ is responsible for similar effective μ terms for the inert generations of Higgs doublets ($\mu_{\beta\gamma} = \lambda_{\alpha\beta\gamma}\langle S_\alpha \rangle$) and for the induced D-fermion masses. Note that we expect all three SM-singlets to develop VEVs, with $\langle S_3 \rangle \ll \langle S_\alpha \rangle$ in order to allow a relatively small μ term (and low fine-tuning) and relatively large exotic \hat{D} and $\hat{\bar{D}}$ particle masses.

As mentioned in the Introduction, in the NMSSM+ instead of \mathbb{Z}_2^{qq} or \mathbb{Z}_2^{lq} we impose either \mathbb{Z}_4^{qq} or \mathbb{Z}_4^{lq} . These are R -symmetries under which the superpotential itself transforms. All terms in whichever of $W_{1,2}$ is allowed by the \mathbb{Z}_4 R -symmetry will be suppressed under an approximate \mathbb{Z}_3^{HD} symmetry, as discussed in the next section and summarised in Table 1. Nonetheless, the suppressed terms in whichever of $W_{1,2}$ is allowed by the R -symmetry will allow the exotic, coloured \hat{D} and $\hat{\bar{D}}$ particles to decay.

3 The NMSSM+

We first define the high energy symmetry and superpotential of the NMSSM+, then give the resulting scale invariant low energy effective theory relevant for phenomenology.

*It has not escaped our attention that this $U(1)_N$ symmetry breaking sector may provide an example of SUSY hybrid inflation with the scalar component of $\hat{\Sigma}$ being the inflaton and those of \hat{S} and $\hat{\bar{S}}$ being the waterfall fields [28, 29].

3.1 The high energy NMSSM+

As discussed above, the high energy NMSSM+ includes the superfield content and gauge group of the E₆SSM (including the $U(1)_N$ gauge group) plus the renormalisable term $\hat{\Sigma}(l\hat{\mathcal{S}}\hat{\bar{\mathcal{S}}} - M_\Sigma^2)$, which spontaneously breaks $U(1)_N$ at the scale $M_\Sigma < M_X$, plus some other non-renormalisable terms, all controlled by a set of symmetries. In this section we give a precise definition of the model, including the non-renormalisable terms and the full set of symmetries.

The full model, valid at a high energy scale just below the GUT breaking scale M_X , is defined by the superfields and symmetries given in Table 1. The resulting gauge invariant high energy superpotential, including important non-renormalisable terms, is given by,

$$\mathcal{W}^{\text{NMSSM+}} = W^{\text{E}_6\text{SSM}} + W^{\mathcal{S}} + \Delta W_0 + \Delta W_1, \quad (16)$$

where $W^{\text{E}_6\text{SSM}}$ is the superpotential of the E₆SSM given in equation (10), while the remaining parts involve the two extra SM-singlet superfields $\hat{\mathcal{S}}$ and $\hat{\bar{\mathcal{S}}}$ with $U(1)_N$ charges Q_S^N and $-Q_S^N$ respectively, where Q_S^N is also the $U(1)_N$ charge of the usual E₆SSM SM-singlets S_i ,

$$\begin{aligned} W^{\mathcal{S}} = & \hat{\Sigma} \left(l\hat{\mathcal{S}}\hat{\bar{\mathcal{S}}} - M_\Sigma^2 \right) \\ & + \frac{b_{ijk}}{M_X^3} \hat{S}_i \hat{S}_j \hat{S}_k \hat{\mathcal{S}}^3 + \frac{d_{ijk}}{M_X} \hat{S}_i \hat{N}_j^c \hat{N}_k^c \hat{\mathcal{S}} \\ & + \left[\frac{c}{M_X^{11}} \hat{\mathcal{S}}^7 \hat{\bar{\mathcal{S}}}^7 + \frac{c'}{M_X^8} \hat{\mathcal{S}}^5 \hat{H}_d \cdot \hat{H}_u \hat{\mathcal{S}}^4 + \frac{c''}{M_X^5} \hat{\mathcal{S}}^3 (\hat{H}_d \cdot \hat{H}_u)^2 \hat{\mathcal{S}} + \dots \right], \end{aligned} \quad (17)$$

$$\Delta W_0 = \frac{r_{ijk}^{udd}}{M_X} \hat{u}_{Ri}^c \hat{d}_{Rj}^c \hat{d}_{Rk}^c \hat{\mathcal{S}} + \frac{r_{ijk}^{DDu}}{M_X} \hat{D}_i \hat{D}_j \hat{u}_{Rk}^c \hat{\mathcal{S}}, \quad (18)$$

$$\Delta W_1 = \frac{r_{ijk}^{SDd}}{M_X} \hat{S}_i \hat{D}_j \hat{d}_{Rk}^c \hat{\mathcal{S}} + \frac{r_{ijk}^{HDq}}{M_X} \hat{H}_{di} \hat{D}_j \hat{Q}_{Lk} \hat{\mathcal{S}}. \quad (19)$$

The rôle of the non-renormalisable terms in $W^{\mathcal{S}}$ is as follows. The VEVs for \mathcal{S} and $\bar{\mathcal{S}}$ at the scale M_Σ , as well as breaking $U(1)_N$, induce desirable cubic SM-singlet terms (required for the NMSSM) through the non-renormalisable couplings proportional to b_{ijk} . The \mathcal{S} and $\bar{\mathcal{S}}$ VEVs also induce SM-singlet couplings to RH neutrinos at low energy controlled by d_{ijk} , however these terms may give negligible contributions to RH neutrino mass compared to other sources of mass. The non-renormalisable terms with 7 conventional low energy fields in the square brackets are important for solving domain wall problems, as explained in subsection 4.3.

The matter parity \mathbb{Z}_2^M actually forbids the non-renormalisable terms in both ΔW_0 and ΔW_1 , but we have included them in order to see the effect of relaxing \mathbb{Z}_2^M , as considered in subsection 4.4. Both these terms yield R -parity violating terms not present in the E₆SSM (since R -parity is automatically conserved by the renormalisable E₆SSM superpotential) once the VEVs for \mathcal{S} and $\bar{\mathcal{S}}$ are inserted. Without \mathbb{Z}_4^{qq} or \mathbb{Z}_4^{lq} other R -parity violating terms would also appear. Depending on which of the two options, namely \mathbb{Z}_4^{qq} or \mathbb{Z}_4^{lq} , is chosen either W_2 is forbidden or both W_1 and ΔW_1 are forbidden, respectively, forbidding rapid proton decay (see subsection 4.4).

	$SU(3)$ rep.	$SU(2)$ rep.	$U(1)_Y$ $\sqrt{5/3}Q^Y$	$U(1)_N$ $\sqrt{40}Q^N$	either \mathbb{Z}_4^{qq} \mathbb{Z}_4^{lq}		optional \mathbb{Z}_2^M	approx. \mathbb{Z}_3^{HD}
\hat{Q}_{Li}	3	2	$+1/6$	+1	$+i$	$+i$	-1	+1
\hat{d}_{Ri}^c	$\bar{3}$	1	$+1/3$	+2	$+i$	$+i$	-1	+1
\hat{u}_{Ri}^c	$\bar{3}$	1	$-2/3$	+1	$+i$	$+i$	-1	+1
\hat{L}_{Li}	1	2	$-1/2$	+2	$-i$	$-i$	-1	+1
\hat{e}_{Ri}^c	1	1	+1	+1	$-i$	$-i$	-1	+1
\hat{N}_i^c	1	1	0	0	$-i$	$-i$	-1	+1
\hat{H}_{d3}	1	2	$-1/2$	-3	$+i$	$+i$	+1	+1
\hat{H}_{u3}	1	2	$+1/2$	-2	$+i$	$+i$	+1	+1
\hat{S}_3	1	1	0	+5	$+i$	$+i$	+1	+1
$\hat{H}_{d\alpha}$	1	2	$-1/2$	-3	$+i$	$+i$	+1	$e^{\frac{2i\pi}{3}}$
$\hat{H}_{u\alpha}$	1	2	$+1/2$	-2	$+i$	$+i$	+1	$e^{\frac{2i\pi}{3}}$
\hat{S}_α	1	1	0	+5	$+i$	$+i$	+1	$e^{\frac{2i\pi}{3}}$
$\hat{\bar{D}}_i$	$\bar{3}$	1	$+1/3$	-3	$+i$	$-i$	+1	$e^{\frac{2i\pi}{3}}$
\hat{D}_i	3	1	$-1/3$	-2	$+i$	$-i$	+1	$e^{\frac{2i\pi}{3}}$
$\hat{\mathcal{S}}$	1	1	0	+5	+1	+1	+1	+1
$\hat{\bar{\mathcal{S}}}$	1	1	0	-5	+1	+1	+1	+1
$\hat{\Sigma}$	1	1	0	0	$-i$	$-i$	+1	+1
\mathcal{W}	1	1	0	0	$-i$	$-i$	+1	+1
$d\theta^2$	1	1	0	0	$+i$	$+i$	+1	+1

Table 1: The gauge group representations and charges and the phase changes under discrete transformations of the superfields and superpotential of the NMSSM+. i labels all three generations whereas α labels the inert generations 1 and 2 only.

3.2 The low energy NMSSM+

The renormalisable part of the low energy effective superpotential which respects the gauge symmetries, the \mathbb{Z}_4 R -symmetry, and matter parity \mathbb{Z}_2^M descending from the high energy theory in equation (16) is then

$$W^{\text{NMSSM}+} = W^{\text{E}_6\text{SSM}} + W_3 = W_0 + W_{1,2} + W_3, \quad (20)$$

where W_0 and $W_{1,2}$ are familiar from the E_6SSM , with $W_{1,2}$ referring to either W_1 or W_2 depending on which option for the R -symmetry is chosen. Once the approximate, generation-dependant \mathbb{Z}_3^{HD} symmetry is imposed W_0 approximately becomes

$$\begin{aligned} W'_0 = & \lambda \hat{S}_3 \hat{H}_d \hat{H}_u + \lambda_{\alpha\beta\gamma} \hat{S}_\alpha \hat{H}_{d\beta} \hat{H}_{u\gamma} + \kappa_{\alpha j k} \hat{S}_\alpha \hat{D}_j \hat{D}_k + h_{3jk}^N \hat{H}_{u3} \hat{L}_{Lj} \hat{N}_k^c \\ & + h_{3jk}^U \hat{H}_{u3} \hat{Q}_{Lj} \hat{u}_{Rk}^c + h_{3jk}^D \hat{H}_{d3} \hat{Q}_{Lj} \hat{d}_{Rk}^c + h_{3jk}^E \hat{H}_{d3} \hat{L}_{Lj} \hat{e}_{Rk}^c. \end{aligned} \quad (21)$$

Note that the Yukawa couplings h other than h_{3jk}^N , h_{3jk}^U , h_{3jk}^D , and h_{3jk}^E are suppressed and that the last four terms above correspond to W_{Yukawa} in equation (5), coupling the active third generation of Higgs doublets to matter. All terms in $W_{1,2}$ are also suppressed under \mathbb{Z}_3^{HD} .

The additional scale invariant term W_3 not present in the E_6SSM is given by

$$W_3 = \frac{k_{ijk}}{3} \hat{S}_i \hat{S}_j \hat{S}_k + \frac{t_{ijk}}{2} \hat{S}_i \hat{N}_j^c \hat{N}_k^c. \quad (22)$$

Imposing the approximate \mathbb{Z}_3^{HD} W_3 approximately becomes

$$W'_3 = \frac{k}{3} \hat{S}^3 + \frac{k_{\alpha\beta\gamma}}{3} \hat{S}_\alpha \hat{S}_\beta \hat{S}_\gamma + \frac{t_{ij}}{2} \hat{S} \hat{N}_i^c \hat{N}_j^c, \quad (23)$$

keeping only the non-suppressed couplings. The term $\frac{t_{ij}}{2} \hat{S} \hat{N}_i^c \hat{N}_j^c$ may be negligible compared to other sources of right-handed neutrino masses (see subsection 4.7).

The low energy NMSSM+ is therefore given approximately as

$$W^{\text{NMSSM}+} \approx W^{\text{E}_6\text{SSM}} + W'_3 \approx W'_0 + W_{1,2} + W'_3. \quad (24)$$

This way of writing the low energy theory shows that the NMSSM+ can be regarded as the E_6SSM *plus* the scale invariant cubic singlet couplings in W'_3 . Note that the low energy superpotential in equation (24) is equivalent to equation (14) under the above approximations. Thus there are two equivalent ways of looking at the NMSSM+, namely either as an extension of the NMSSM by the inclusion of an exotic sector, or as an extension of the E_6SSM by the inclusion of cubic singlet terms (with the understanding that the $U(1)_N$ is broken at high energies).

4 Aspects of the NMSSM+

In this section we discuss some interesting theoretical and phenomenological aspects of the NMSSM+. We give a commentary concerning the different symmetries of the high energy theory and show how they result in the low energy NMSSM+ described in the previous section.

4.1 Discussion of the \mathbb{Z}_4 global R -symmetry

In the NMSSM+ instead of \mathbb{Z}_2^{qq} or \mathbb{Z}_2^{lq} we impose either \mathbb{Z}_4^{qq} or \mathbb{Z}_4^{lq} . These are R -symmetries under which the superfields and indeed the superpotential itself transform as follows in each case:

$$\begin{aligned}
& \mathbb{Z}_4^{qq} \\
(\hat{S}, \hat{H}_d, \hat{H}_u, \hat{\hat{D}}, \hat{D}, \hat{Q}_L, \hat{u}_R^c, \hat{d}_R^c) & \rightarrow e^{\frac{i\pi}{2}} (\hat{S}, \hat{H}_d, \hat{H}_u, \hat{\hat{D}}, \hat{D}, \hat{Q}_L, \hat{u}_R^c, \hat{d}_R^c), \\
(\hat{L}_L, \hat{e}_R^c, \hat{N}^c) & \rightarrow e^{\frac{3i\pi}{2}} (\hat{L}_L, \hat{e}_R^c, \hat{N}^c), \\
(\hat{\mathcal{S}}, \hat{\hat{\mathcal{S}}}) & \rightarrow (\hat{\mathcal{S}}, \hat{\hat{\mathcal{S}}}), \\
\hat{\Sigma} & \rightarrow e^{\frac{3i\pi}{2}} \hat{\Sigma}, \\
\mathcal{W} & \rightarrow e^{\frac{3i\pi}{2}} \mathcal{W} \\
\left[\Rightarrow \quad d\theta^2 \quad \rightarrow \quad e^{\frac{-3i\pi}{2}} d\theta^2 = e^{\frac{i\pi}{2}} d\theta^2 \right]. & \tag{25}
\end{aligned}$$

$$\begin{aligned}
& \mathbb{Z}_4^{lq} \\
(\hat{S}, \hat{H}_d, \hat{H}_u, \hat{Q}_L, \hat{u}_R^c, \hat{d}_R^c) & \rightarrow e^{\frac{i\pi}{2}} (\hat{S}, \hat{H}_d, \hat{H}_u, \hat{Q}_L, \hat{u}_R^c, \hat{d}_R^c), \\
(\hat{\hat{D}}, \hat{D}, \hat{L}_L, \hat{e}_R^c, \hat{N}^c) & \rightarrow e^{\frac{3i\pi}{2}} (\hat{\hat{D}}, \hat{D}, \hat{L}_L, \hat{e}_R^c, \hat{N}^c), \\
(\hat{\mathcal{S}}, \hat{\hat{\mathcal{S}}}) & \rightarrow (\hat{\mathcal{S}}, \hat{\hat{\mathcal{S}}}), \\
\hat{\Sigma} & \rightarrow e^{\frac{3i\pi}{2}} \hat{\Sigma}, \\
\mathcal{W} & \rightarrow e^{\frac{3i\pi}{2}} \mathcal{W} \\
\left[\Rightarrow \quad d\theta^2 \quad \rightarrow \quad e^{\frac{-3i\pi}{2}} d\theta^2 = e^{\frac{i\pi}{2}} d\theta^2 \right]. & \tag{26}
\end{aligned}$$

The renormalisable high energy superpotential with $\mathbb{Z}_4^{qq,lq}$ is then

$$\mathcal{W}_{\text{ren}}^{\text{NMSSM}+} = W_0 + W_{1,2} + \hat{\Sigma} \left(l \hat{\mathcal{S}} \hat{\hat{\mathcal{S}}} - M_{\Sigma}^2 \right), \tag{27}$$

which includes only the renormalisable terms of $\mathcal{W}^{\text{NMSSM}+}$ in equation (16). We have thus found R -symmetries that allow the usual trilinear superpotential terms of the $E_6\text{SSM}$, avoid rapid proton decay by forbidding either W_1 or W_2 , and also distinguish between the conventional SM-singlets S_i and the new \mathcal{S} in a way that allows for the Σ scale and the EWSB scale to be naturally separated even though S_i and \mathcal{S} share the same gauge charges. The further consequences of these R -symmetries are discussed below.

4.2 High energy $U(1)_N$ gauge symmetry breaking in the NMSSM+

At the Σ scale there is no radiative EWSB. The high energy scalar potential relevant for finding the VEVs generated at the Σ scale is obtained from the renormalisable terms in

$W^{\mathcal{S}}$ in equation (17), together with D -terms and soft mass terms,

$$\begin{aligned} \mathcal{V} = & |l\mathcal{S}\bar{\mathcal{S}} - M_{\Sigma}^2|^2 \\ & + l^2|\Sigma|^2 (|\mathcal{S}|^2 + |\bar{\mathcal{S}}|^2) + \frac{D_N^2}{2} \\ & + m_{\mathcal{S}}^2|\mathcal{S}|^2 + m_{\bar{\mathcal{S}}}^2|\bar{\mathcal{S}}|^2 + m_{\Sigma}^2|\Sigma|^2 \\ & + \text{terms with zero VEV}, \end{aligned} \quad (28)$$

where $m_{(\mathcal{S}, \bar{\mathcal{S}}, \Sigma)}^2$ are soft supersymmetry breaking masses-squared and

$$D_N = g'_1 \left(Q_S^N |\mathcal{S}|^2 - Q_{\bar{S}}^N |\bar{\mathcal{S}}|^2 + \sum_a Q_a^N |\varphi_a|^2 \right) \quad (29)$$

is the $U(1)_N$ D -term, with φ_a and Q_a^N the other scalars (the E₆SSM-like scalars that survive to low energy) and their $U(1)_N$ charges. Minimising the scalar potential with respect to \mathcal{S} , $\bar{\mathcal{S}}$, and Σ gives

$$|\langle \mathcal{S} \rangle|^2 = \frac{M_{\Sigma}^2}{l} + \mathcal{O}(m_{\mathcal{S}}^2, m_{\bar{\mathcal{S}}}^2), \quad (30)$$

$$|\langle \bar{\mathcal{S}} \rangle|^2 = \frac{M_{\Sigma}^2}{l} + \mathcal{O}(m_{\mathcal{S}}^2, m_{\bar{\mathcal{S}}}^2), \quad (31)$$

$$\langle \Sigma \rangle = 0, \quad (32)$$

$$\langle D_N \rangle = \frac{m_{\bar{\mathcal{S}}}^2 - m_{\mathcal{S}}^2}{2g'_1 Q_S^N}. \quad (33)$$

The first term in the potential sets the scale for the \mathcal{S} and $\bar{\mathcal{S}}$ VEVs to M_{Σ} and the form of the D -term requires them to be equal to each other up to possible corrections of order the soft supersymmetry breaking scale. This is why we include the *pair* of extra fields with opposite $U(1)_N$ charges, so that the minimisation conditions lead to $U(1)_N$ breaking that is approximately D -flat.

The VEV for the $U(1)_N$ D -term, proportional to $m_{\bar{\mathcal{S}}}^2 - m_{\mathcal{S}}^2$, may be small, or even zero, depending on the nature of supersymmetry breaking and its mediation to the visible sector. Below the Σ scale, when the fields that acquire masses are integrated out, any soft scale $\langle D_N \rangle$ will just lead to corrections for the effective soft supersymmetry breaking masses-squared for the other scalars

$$\Delta m_a^2 = \frac{Q_a^N}{Q_S^N} (m_{\bar{\mathcal{S}}}^2 - m_{\mathcal{S}}^2), \quad (34)$$

as discussed in [30].

4.3 How the NMSSM+ solves the domain wall problem

If M_{Σ} is not too far below the unification scale M_X scale then an effective NMSSM-like $\hat{\mathcal{S}}^3$ term is generated by the first non-renormalisable term in $W^{\mathcal{S}}$ in equation (17),

$$\frac{b_{333}}{M_X^3} \hat{\mathcal{S}}^3 \langle \hat{\mathcal{S}}^3 \rangle, \quad (35)$$

with a coefficient $k/3$ not too far from unity. This will break the otherwise present global $U(1)$ symmetry of the effective low energy theory below the Σ scale down to a global \mathbb{Z}_3 [†].

4.3.1 Domain wall destabilisation

The discrete R -symmetry of the NMSSM+ also allows the non-renormalisable terms in square brackets in equation (17),

$$\frac{c}{M_X^{11}} \hat{S}^7 \hat{\bar{S}}^7 + \frac{c'}{M_X^8} \hat{S}^5 \hat{H}_d \cdot \hat{H}_u \hat{\bar{S}}^4 + \frac{c''}{M_X^5} \hat{S}^3 (\hat{H}_d \cdot \hat{H}_u)^2 \hat{\bar{S}} \quad (36)$$

to appear in the high energy superpotential. When the high scale $\bar{\mathcal{S}}$ VEV is inserted these terms generate the effective dimension 7 terms \hat{S}^7 , $\hat{S}^5(\hat{H}_d \cdot \hat{H}_u)$, and $\hat{S}^3(\hat{H}_d \cdot \hat{H}_u)^2$, each suppressed by 4 powers of M_X , in the low energy superpotential. Through 4-loop tadpoles of the form in Figure 1 these dimension 7 terms and the dimension 3 terms will generate linear terms in the potential of order [33]

$$\frac{1}{(16\pi^2)^4} m_{\text{soft}}^3 (S + S^*), \quad (37)$$

breaking the accidental global \mathbb{Z}_3 and destabilising its cosmological domain walls.

4.3.2 Forbidden non-renormalisable operators and lack of divergences

It is important that the above terms are allowed by the R -symmetry. As well as these terms containing 7 low energy (conventional E_6 SSM) superfields, the R -symmetry of the NMSSM+ also allows terms containing 5 low energy superfields (where at least one superfield is a lepton or exotic coloured superfield). Non-renormalisable terms containing just 3 low energy superfields are discussed in the following subsection along with Kähler potential terms containing just two low energy superfields.

Superpotential terms containing even numbers of low energy superfields and Kähler potential terms containing odd numbers of low energy superfields are completely forbidden by the R -symmetry. This is important because such terms would induce effective terms in the low energy superpotential that in the NMSSM have been shown to lead to dangerously divergent tadpoles [33].

4.4 Matter parity violating operators

For completeness we list the following matter parity violating terms which are allowed by the gauge symmetry ($\forall i, j, k$) but are forbidden by either \mathbb{Z}_4^{qq} or \mathbb{Z}_4^{lq} in the superpotential:

$$\begin{aligned} \hat{L}_{Li} \hat{L}_{Lj} \hat{e}_{Rk}^c \frac{\hat{\bar{S}}}{M_X}, \quad \hat{Q}_{Li} \hat{L}_{Lj} \hat{d}_{Rk}^c \frac{\hat{\bar{S}}}{M_X}, \quad \hat{H}_{di} \hat{H}_{dj} \hat{e}_{Rk}^c \frac{\hat{\bar{S}}}{M_X}, \\ \hat{S}_i \hat{H}_{uj} \hat{L}_{Lk} \frac{\hat{\bar{S}}}{M_X}, \quad \hat{N}_i^c \hat{H}_{dj} \hat{H}_{uk} \frac{\hat{\bar{S}}}{M_X}; \end{aligned} \quad (38)$$

[†] This $U(1)$ symmetry is familiar in the NMSSM in the limit of vanishing S^3 term, and the same symmetry in the conventional E_6 SSM is gauged to become the $U(1)_N$ symmetry. Here the $U(1)_N$ gauge symmetry is spontaneously broken by the high scale $\bar{\mathcal{S}}$ VEV, resulting at the low energy scale in an effective \hat{S}^3 term in the superpotential.

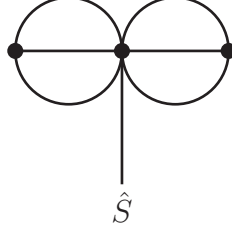


Figure 1: The form of SM-singlet tadpoles caused by the existence of dimension 7 terms in the superpotential as well as dimension 3 terms.

and in the Kähler potential:

$$\begin{aligned} \hat{L}_{Li} \hat{H}_{dj}^\dagger \frac{\hat{S}}{M_X} + \text{c.c.}, \quad & \hat{L}_{Li} \hat{H}_{dj}^\dagger \frac{\hat{S}^\dagger}{M_X} + \text{c.c.}, \\ \hat{N}_i^c \hat{S}_j^\dagger \frac{\hat{S}}{M_X} + \text{c.c.}, \quad & \hat{N}_i^c \hat{S}_j^\dagger \frac{\hat{S}^\dagger}{M_X} + \text{c.c.} \end{aligned} \quad (39)$$

Since we assume either \mathbb{Z}_4^{qq} or \mathbb{Z}_4^{lq} , none of the above terms are present.

On the other hand, the following matter parity violating terms appearing in ΔW_1 in equation (19) are allowed by the gauge symmetry and \mathbb{Z}_4^{qq} , but are forbidden by \mathbb{Z}_4^{lq} :

$$\hat{S}_i \hat{D}_j \hat{d}_{Rk}^c \frac{\hat{S}}{M_X}, \quad \hat{H}_{di} \hat{D}_j \hat{Q}_{Lk} \frac{\hat{S}}{M_X}; \quad (40)$$

and the following matter parity violating terms appearing in ΔW_0 in equation (18) are allowed by the gauge symmetry and both \mathbb{Z}_4^{qq} and \mathbb{Z}_4^{lq} :

$$\hat{u}_{Ri}^c \hat{d}_{Rj}^c \hat{d}_{Rk}^c \frac{\hat{S}}{M_X}, \quad \hat{D}_i \hat{D}_j \hat{u}_{Rk}^c \frac{\hat{S}}{M_X}. \quad (41)$$

The following matter parity violating Kähler potential terms ΔK_1 are also allowed by the gauge symmetry and \mathbb{Z}_4^{qq} , but forbidden by \mathbb{Z}_4^{lq} :

$$\Delta K_1 = \frac{r_{ij}^{dD}}{M_X} \hat{d}_{Ri}^c \hat{D}_j^\dagger \hat{S} + \frac{r_{ij}^{dD'}}{M_X} \hat{d}_{Ri}^c \hat{D}_j^\dagger \hat{S}^\dagger + \text{c.c.} \quad (42)$$

These terms are all forbidden by imposing the matter parity \mathbb{Z}_2^M , but we list them in case one wishes to relax \mathbb{Z}_2^M . We give the two cases, for the two R -parity choices, below.

4.4.1 \mathbb{Z}_4^{qq} without \mathbb{Z}_2^M

In this case ΔW_0 , ΔW_1 , and ΔK_1 are all allowed and the low energy superpotential is given by

$$W^{\text{NMSSM}+} = W_0 + W_1 + W_3 + \Delta W_0 + \Delta W_1. \quad (43)$$

The \hat{D} and $\hat{\bar{D}}$ are interpreted as anti-diquarks and diquarks respectively so that $W_0 + W_1$ respects B and L (baryon and lepton number) conservation. In this case ΔW_0 , ΔW_1 , and

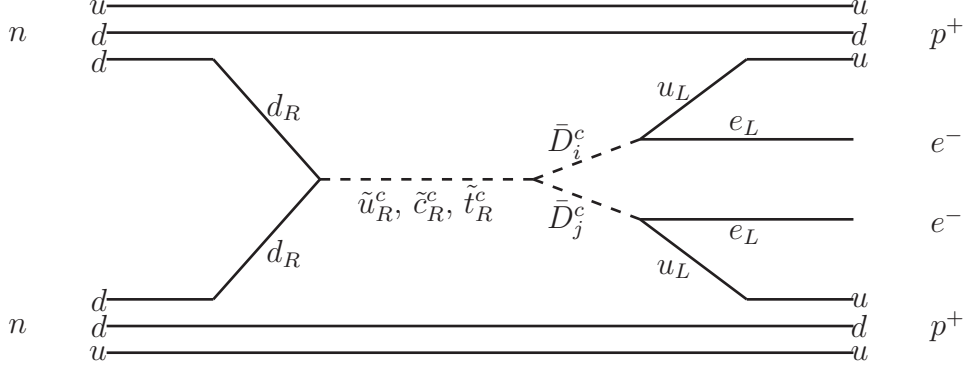


Figure 2: A Feynman diagram for neutrinoless double beta decay in the theory where \mathbb{Z}_4^{lq} is imposed without matter parity \mathbb{Z}_2^M . The two couplings involving squarks are from the two matter parity violating terms in ΔW_0 . Here they are used together to change lepton number by two while preserving baryon number. The first vertex is at least CKM suppressed since r_{ijk}^{udd} from equation (18) has to be antisymmetric in its last two indices (jk) in the gauge basis (due to $SU(3)$ gauge symmetry) and therefore cannot couple two down quark interaction states together. The second vertex is from the soft A term associated with r_{ijk}^{DDu} .

ΔK_1 break B , but respect L [‡]. It is clear that since L is conserved, proton decay with one lepton in the final state is forbidden. More generally, since *every one* of the matter parity violating couplings, coming from ΔW_0 , ΔW_1 , and ΔK_1 , change B by exactly one, $\Delta B = -1$ proton decay is forbidden as long as having any supersymmetric particles in the final state is kinematically not allowed.

Rapid proton decay is therefore avoided, but the LSP would be allowed to decay; for example, the effective superpotential term $\hat{u}_{Ri}^c \hat{d}_{Rj}^c \hat{d}_{Rk}^c$ allows the decays $\tilde{\chi}^0 \rightarrow p^\pm K^\mp$ if the neutralino $\tilde{\chi}^0$ contains some admixture of the bino (\tilde{B}), the superpartner of the Abelian hypercharge gauge boson.

While we observe that rapid proton decay is avoided in this scenario, certainly as long as the lightest supersymmetric particle is heavier than the proton, $\Delta B = \pm 2$ effects [34], such as $n-\bar{n}$ oscillations [35] and dinucleon decay $p^+ p^+ \rightarrow K^+ K^+$, would also need to be considered.

4.4.2 \mathbb{Z}_4^{lq} without \mathbb{Z}_2^M

In this case ΔW_0 , but neither ΔW_1 nor ΔK_1 , is allowed and

$$W^{\text{NMSSM}+} = W_0 + W_2 + W_3 + \Delta W_0, \quad (44)$$

where

$$W_2 = g_{ijk}^N \hat{N}_i^c \hat{D}_j \hat{d}_{Rk}^c + g_{ijk}^E \hat{D}_i \hat{u}_{Rj}^c \hat{e}_{Rk}^c + g_{ijk}^D \hat{D}_i \hat{Q}_{Lj} \hat{L}_{Lk}. \quad (45)$$

Without ΔW_0 , just looking at $W_0 + W_2$, \hat{D} and $\hat{\bar{D}}$ would be interpreted as leptoquarks and anti-leptoquarks respectively, implying B and L conservation. The presence of ΔW_0

[‡]RH neutrinos violate lepton number conservation via the term in W_3 and via any large intermediate scale Majorana mass term (see subsection 4.7), both leading to $\Delta L = \pm 2$ effects. This does not affect the arguments of this subsection

introduces the terms $\hat{\bar{D}}_i \hat{\bar{D}}_j \hat{u}_{Rk}^c$ and $\hat{u}_{Ri}^c \hat{d}_{Rj}^c \hat{d}_{Rk}^c$. With the above interpretation these terms have $(\Delta B, \Delta L) = (-1, -2)$ and $(-1, 0)$ respectively. Since $\Delta L = \pm 1$ terms are absent, proton decay with one lepton in the final state is again forbidden. More generally, in this scenario too, every one of the matter parity violating couplings, coming from ΔW_0 , change B by exactly one, meaning that once again $\Delta B = -1$ proton decay is forbidden without supersymmetric particles in the final state.

Once again $\Delta B = \pm 2$ effects would have to be considered, but now so too would $\Delta L = \pm 2$ effects. For instance, both of the matter parity violating terms from ΔW_0 can be combined with $\hat{\bar{D}}_i \hat{Q}_{Lj} \hat{L}_{Lk}$ from W_2 to give contributions to neutrinoless double beta decay, as shown in Figure 2, different from those considered in [36].

4.5 Harmless non-renormalisable terms

For each of the superpotential and Kähler potential terms that we have mentioned so far that are invariant with respect to the gauge symmetry and the \mathbb{Z}_4 global R -symmetry and optionally matter parity there is also the same term multiplied by any arbitrary power of $(\mathcal{S}\bar{\mathcal{S}})/M_X^2$ with some coefficient. Assuming that

$$\langle \mathcal{S}\bar{\mathcal{S}} \rangle \approx \frac{M_\Sigma^2}{l}$$

is somewhat less than M_X^2 these higher order terms will be sub-dominant in the perturbative expansion of the low energy theory, and represent harmless corrections to the leading terms.

4.6 The suppression of flavour changing neutral currents and the generation of exotic D-fermion masses

We now motivate the approximate \mathbb{Z}_3^{HD} given in Table 1 as a way to remove flavour changing neutral currents and allow the generation of D-fermion masses from inert SM-singlet VEVs that can naturally be at a scale slightly larger than the EWSB scale.

Let us examine the Yukawa couplings present in the low energy NMSSM+ potential given in equation (20)

$$W^{\text{NMSSM+}} = W_0 + W_{1,2} + W_3. \quad (46)$$

W_0 contains Yukawa couplings of all three generations of Higgs doublets to matter as well as

$$\lambda_{ijk} \hat{S}_i \hat{H}_{dj} \hat{H}_{uk} + \kappa_{ijk} \hat{S}_i \hat{\bar{D}}_j \hat{\bar{D}}_k.$$

Note that SM-singlet VEVs are the only way for the exotic D-fermions to acquire mass (the scalar components also acquire mass through soft supersymmetry breaking terms).

In the NMSSM+ we propose that the inert SM-singlets S_α acquire VEVs and that these VEVs are responsible for the D-fermion masses, rather than the active SM-singlet VEV $s = s_3$, where we define $\alpha, \beta, \gamma \in \{1, 2\}$ and $s_i = \sqrt{2} \langle S_i \rangle$. It is important that the $\hat{\bar{D}}$ and \hat{D} receive mass from different singlets to those responsible for the μ term since

otherwise their mass limits would lead to larger values of μ and hence to fine-tuning. By contrast, the VEVs of the other (inert) SM-singlets S_α may be slightly larger than the EWSB scale, providing large exotic masses, without necessarily introducing fine-tuning. This would not be advantageous in the conventional E_6 SSM, since any and all of the three possible SM-singlet VEVs would contribute to $\langle D_N \rangle$ and any large SM-singlet VEV would therefore lead to fine-tuning, as explained in subsection 5.1.

In the conventional E_6 SSM the approximate flavour symmetry \mathbb{Z}_2^H , under which only the third (active) generation of Higgs doublets and SM-singlets are even, is applied to suppress the Yukawa couplings of the inert generations of Higgs doublets to matter, suppressing flavour changing neutral currents and explaining why only the active generation radiatively acquires VEVs. In the NMSSM+ something different is required to do this job since \mathbb{Z}_2^H would suppress couplings of the form $\kappa_{\alpha ij}$. In the NMSSM+ these couplings should be large both to explain how large inert singlet VEVs could be radiatively induced and because these couplings then appear in the masses induced for the D-fermions by those inert singlet VEVs. Furthermore, in order to keep a slight hierarchy $s_\alpha \gg s_3$ natural terms coupling the EWSB scale VEVs s_3 , v_d , and v_u to the larger VEVs s_2 and s_1 should not be very large. We therefore also require an approximate flavour symmetry that suppresses trilinear SM-singlet couplings k_{ijk} from W_3 other than $k = k_{333}$ and those of the form $k_{\alpha\beta\gamma}$, as well as various λ_{ijk} couplings, namely those of the forms $\lambda_{\alpha 3i}$ and $\lambda_{\alpha i 3}$ [§].

In the NMSSM+, instead of \mathbb{Z}_2^H , we have an approximate symmetry \mathbb{Z}_3^{HD} under which

$$\mathbb{Z}_3^{HD} \quad (\hat{S}_\alpha, \hat{H}_{d\alpha}, \hat{H}_{u\alpha}, \hat{D}_i, \hat{D}_i) \rightarrow e^{\frac{2i\pi}{3}} (\hat{S}_\alpha, \hat{H}_{d\alpha}, \hat{H}_{u\alpha}, \hat{D}_i, \hat{D}_i) \quad (47)$$

and no other superfields transform. This then leads to the approximate NMSSM+ superpotential given in equation (14). Like \mathbb{Z}_2^H , this approximate symmetry suppresses Yukawa couplings of the inert generations of Higgs doublets to matter and all couplings in $W_{1,2}$, suppressing flavour changing neutral currents. It also suppresses all k_{ijk} couplings other than $k = k_{333}$ and those of the form $k_{\alpha\beta\gamma}$; all λ_{ijk} couplings other than $\lambda = \lambda_{333}$ and those of the form $\lambda_{\alpha\beta\gamma}$; and all κ_{ijk} couplings other than those of the form $\kappa_{\alpha jk}$.

We emphasise that the \mathbb{Z}_3^{HD} symmetry should not be exact. An exact \mathbb{Z}_3^{HD} symmetry would exactly forbid both W_1 and W_2 , meaning that the exotic coloured \hat{D} and $\hat{\bar{D}}$ particles would not be able to decay (the same reason why \mathbb{Z}_2^H cannot be exact in the E_6 SSM.) Furthermore, since the inert SM-singlets transform under the symmetry the exact symmetry would be spontaneously broken by the inert SM-singlet VEVs and this would lead to cosmological domain walls. This symmetry is therefore regarded as an approximate flavour symmetry, on the same footing as the approximate \mathbb{Z}_2^H in the E_6 SSM. Since the symmetry is not exact small effective linear terms for the inert SM-singlets, which break \mathbb{Z}_3^{HD} , will be generated for the inert SM-singlets in the same way that they are generated for the active SM-singlet, via tadpole diagrams of the form in Figure 1 in subsection 4.3.

[§]We note that these are the “f- and z-couplings” that are required to be exactly zero in the EZSSM dark matter scenario [32]. In this version of the model a discrete symmetry \mathbb{Z}_2^S is imposed under which only \hat{S}_α are odd. Unfortunately this symmetry would also forbid $\kappa_{\alpha ij}$ which should be large in the NMSSM+ for the reasons described.

4.7 Right-handed neutrino masses

It is important to note that the \mathbb{Z}_4 R -symmetry forbids Planck scale Majorana RH neutrino masses which are otherwise allowed since RH neutrinos are complete gauge singlets.

Once the \mathbb{Z}_3^{HD} symmetry from the previous subsection has been applied W_3 from equation (22) approximately becomes

$$W'_3 = \frac{k}{3}\hat{S}^3 + \frac{k_{\alpha\beta\gamma}}{3}\hat{S}_\alpha\hat{S}_\beta\hat{S}_\gamma + \frac{t_{ij}}{2}\hat{S}\hat{N}_i^c\hat{N}_j^c \quad (48)$$

as in equation (23). A Majorana RH neutrino mass is therefore generated by the active EWSB scale SM-singlet VEV $\langle S \rangle = s_3/\sqrt{2}$.

However, some extra mechanism would have to be responsible for generating an intermediate scale Majorana mass as needed for a type-I see-saw mechanism. This could be achieved, for example, by a complete gauge singlet \hat{S}_N that transforms under the R -symmetry as

$$\hat{S}_N \rightarrow e^{\frac{i\pi}{2}}\hat{S}_N = +i\hat{S}_N \quad (49)$$

and couples to the RH neutrinos via the superpotential term

$$\frac{e_{ij}}{2}\hat{S}_N\hat{N}_i^c\hat{N}_j^c, \quad (50)$$

with S_N acquiring an appropriate intermediate scale VEV.

4.8 Grand unification

The situation with respect to grand unification is similar to that in the E_6 SSM. The extra $SU(2)$ doublets H' and \bar{H}' mentioned in the Introduction, which are included in the E_6 SSM [20] for gauge coupling unification, may be included with a mass of order 10 TeV.

Alternatively in the NMSSM+ the H' and \bar{H}' may be omitted. Without these fields and with the E_6 assumed broken directly to $G_{\text{SM}} \otimes U(1)_N$ at the GUT scale, the renormalisation group equations do not actually allow the gauge couplings to unify below the Planck scale. However, since in the NMSSM+ we already have an intermediate scale M_Σ slightly below the assumed grand unification scale, one could assume that something like in the Minimal E_6 SSM [31] happens, where E_6 is not broken directly to $G_{\text{SM}} \otimes U(1)_N$ and where the running is modified above some intermediate scale.

Whichever scenario is chosen, the fact that the matter content consists of complete 27 representations of E_6 (plus \mathcal{S} , $\bar{\mathcal{S}}$, Σ , and possibly H' and \bar{H}') ensures anomaly cancellation from the GUT scale to the Σ scale where $U(1)_N$ is broken.

5 Fine-Tuning

In this section we give a qualitative discussion of the tree-level fine-tuning which is present in the E_6 SSM, and then show how the NMSSM+ leads to a dramatic improvement in tree-level fine-tuning.

5.1 Tree-level fine-tuning in the conventional E_6 SSM

The E_6 SSM active scalar potential relevant for EWSB may be written [18]

$$\begin{aligned} V = & \lambda^2 |S|^2 (|H_d|^2 + |H_u|^2) + \lambda^2 |H_d \cdot H_u|^2 \\ & + \frac{g_1^2}{8} (|H_d|^2 - |H_u|^2)^2 + \frac{g_2^2}{8} (H_d^\dagger \sigma^a H_d - H_u^\dagger \sigma^a H_u)^2 + \frac{D_N^2}{2} \\ & + m_d^2 |H_d|^2 + m_u^2 |H_u|^2 + m_S^2 |S|^2 + [\lambda A_\lambda S H_d H_u + \text{c.c.}], \end{aligned} \quad (51)$$

where

$$D_N = g_1' (Q_d |H_d|^2 + Q_u |H_u|^2 + Q_S |S|^2 + \text{terms with zero VEV}) \quad (52)$$

is the $U(1)_N$ D -term.

The Higgs tree-level minimisation conditions in this model are [18]

$$\begin{aligned} \frac{\partial V}{\partial s} &= m_S^2 s - \frac{\lambda A_\lambda}{\sqrt{2}} v_1 v_2 + \frac{\lambda^2}{2} (v_d^2 + v_u^2) s \\ &\quad + \frac{g_1'^2}{2} (Q_d v_d^2 + Q_u v_u^2 + Q_S s^2) Q_S s = 0, \\ \frac{\partial V}{\partial v_d} &= m_d^2 v_d - \frac{\lambda A_\lambda}{\sqrt{2}} s v_u + \frac{\lambda^2}{2} (v_u^2 + s^2) v_d + \frac{\bar{g}^2}{8} (v_d^2 - v_u^2) v_d \\ &\quad + \frac{g_1'^2}{2} (Q_d v_d^2 + Q_u v_u^2 + Q_S s^2) Q_d v_d = 0, \\ \frac{\partial V}{\partial v_u} &= m_u^2 v_u - \frac{\lambda A_\lambda}{\sqrt{2}} s v_d + \frac{\lambda^2}{2} (v_d^2 + s^2) v_u + \frac{\bar{g}^2}{8} (v_u^2 - v_d^2) v_u \\ &\quad + \frac{g_1'^2}{2} (Q_d v_d^2 + Q_u v_u^2 + Q_S s^2) Q_u v_u = 0, \end{aligned} \quad (53)$$

where $\bar{g} = \sqrt{g_2^2 + g'^2}$, $s = \sqrt{2} \langle S \rangle$, and $v_{d,u} = \sqrt{2} \langle H_{d,u} \rangle$.

We begin by dropping factors of order unity and making the approximations $s \gg v_d, v_u$ and $\lambda \ll \bar{g}$, and writing $v \sim v_d \sim v_u$, and $m_d^2 \sim m_u^2 \sim m^2$. These equations may then be combined to yield,

$$M_{Z'}^2 \sim \mu A_\lambda - m^2 + \mu^2 - M_{Z'}^2, \quad (54)$$

where $\mu \sim \lambda s$ and $M_{Z'}^2 \approx -2m_S^2$. More accurate minimisation conditions will be derived below, keeping all the factors of order unity. However equation (54) is sufficient to show that in order to avoid *tree-level* fine-tuning we need to keep both μ and $M_{Z'}$ as close to the electroweak scale as possible. Clearly the recent experimental limit of $M_{Z'} > 2$ TeV [27] leads to significant fine-tuning. We emphasise that the appearance of $M_{Z'}$ in the tree-level minimisation condition is characteristic of all Z' models where the usual Higgs doublets carry $U(1)'$ charges (e.g. it applies to all E_6 models but not the $U(1)_{B-L}$ model).

For the more accurate conditions we instead minimise with respect to s^2 and $v^2 = v_d^2 + v_u^2$. Classically minimising with respect to s^2 yields

$$\frac{\partial V}{\partial (s^2)} = 2g_1'^2 Q_S^2 s^2 + 4\lambda^2 (v_d^2 + v_u^2) + 2m_S^2 + 4g_1'^2 Q_S (Q_d v_d^2 + Q_u v_u^2) + 2\sqrt{2}\lambda A_\lambda \frac{v_d v_u}{s} = 0.$$

The limit on $M_{Z'}^2 \approx 2g_1'^2 Q_S^2 s^2$ implies that $s^2 \gg v^2$ and this then requires a large negative m_S^2 . The equation approximately becomes

$$g_1'^2 Q_S^2 s^2 + m_S^2 \approx 0 \Rightarrow M_{Z'}^2 \approx -2m_S^2. \quad (55)$$

Classically minimising with respect to v^2 yields

$$\begin{aligned}\frac{\partial V}{\partial(v^2)} &= \bar{g}^2(c_\beta^2 - s_\beta^2)^2 v^2 + 4g_1'^2 (Q_d c_\beta^2 + Q_u s_\beta^2)^2 v^2 + 8\lambda^2 s_\beta^2 c_\beta^2 v^2 \\ &\quad + 4\lambda^2 s^2 + 2m_d^2 c_\beta^2 + 2m_u^2 s_\beta^2 \\ &\quad + 4g_1'^2 Q_S s^2 (Q_d c_\beta^2 + Q_u s_\beta^2) + 4\sqrt{2}\lambda A_\lambda s c_\beta s_\beta = 0.\end{aligned}\quad (56)$$

In order to satisfy this condition with $s^2 \gg v^2$ fine-tuning is required to occur between the s^2 terms, namely $\lambda^2 s^2$ and $g_1'^2 Q_S s^2 (Q_d c_\beta^2 + Q_u s_\beta^2)$, where the latter term is proportional to the Z' mass squared.

5.2 Tree-level fine-tuning in the NMSSM+

Imposing either \mathbb{Z}_4^{qq} or \mathbb{Z}_4^{lq} and the approximate \mathbb{Z}_3^{HD} leads to the low energy NMSSM+ in equation (14) which in turn leads to the following scalar potential, where we only include fields able to acquire VEVs, namely $H_{(d,u)3}$ and S_i ,

$$\begin{aligned}V \approx & \lambda^2 |S|^2 (|H_d|^2 + |H_u|^2) + |\lambda H_d H_u + k_{333} S_3^2|^2 \\ & + |k_{222} S_2^2 + 2k_{[221]} S_2 S_1 + k_{[211]} S_1^2|^2 \\ & + |k_{111} S_1^2 + 2k_{[112]} S_1 S_2 + k_{[122]} S_2^2|^2 \\ & + \frac{g_1^2}{8} (|H_d|^2 - |H_u|^2)^2 + \frac{g_2^2}{8} (H_d^\dagger \sigma^a H_d - H_u^\dagger \sigma^a H_u)^2 \\ & + \left[\lambda A_\lambda S_3 H_d \cdot H_u + \frac{k_{333} A_{k_{333}}}{3} S_3^3 + \sum_{\alpha, \beta, \gamma} \frac{k_{\alpha\beta\gamma} A_{k_{\alpha\beta\gamma}}}{3} S_\alpha S_\beta S_\gamma + \text{c.c.} \right] \\ & + m_{S_3}^2 |S_3|^2 + m_{S_2}^2 |S_2|^2 + m_{S_1}^2 |S_1|^2 + m_d^2 |H_d|^2 + m_u^2 |H_u|^2.\end{aligned}\quad (57)$$

Classically minimising V with respect to s_3^2 yields

$$\begin{aligned}\left\langle \frac{\partial V}{\partial(s^2)} \right\rangle = 0 \approx & 4\lambda^2 v^2 + 2m_S^2 + 8k (\lambda v_d v_u + k s^2) \\ & + 2\sqrt{2}\lambda A_\lambda \frac{v_d v_u}{s} + 2\sqrt{2}k A_k s,\end{aligned}\quad (58)$$

where $k = k_{333}$ and $s = s_3$, and classically minimising with respect to v^2 yields

$$\begin{aligned}\left\langle \frac{\partial V}{\partial(v^2)} \right\rangle = 0 \approx & \bar{g}^2 (\cos^2 \beta - \sin^2 \beta)^2 v^2 + 8\lambda \sin \beta \cos \beta (\lambda \sin \beta \cos \beta v^2 + k s^2) \\ & + 4\lambda^2 s^2 + 2m_d^2 \cos^2 \beta + 2m_u^2 \sin^2 \beta \\ & + 4\sqrt{2}\lambda A_\lambda s \cos \beta \sin \beta.\end{aligned}\quad (59)$$

By design of \mathbb{Z}_3^{HD} these conditions are approximately independent of s_2 and s_1 . There is no tree-level fine-tuning required for these minimisation conditions to yield EW scale VEVs since the Z' mass does not appear and the active singlet VEV s may be taken to be low, since it is unrelated to the exotic fermion masses, with λ reasonably large in order to yield a large correction to the Higgs boson tree-level mass, so that the effective μ term is not too large. As mentioned in the Introduction, we also expect this model to exhibit less

fine-tuning overall than the NMSSM due to the presence of the extra matter which allows for larger values of λ without violating perturbation theory up to the GUT scale. The Higgs mass equation (2) is again relevant, but the presence of extra matter should allow greater values for λ to be perturbative up to high scales (see e.g. [17]), increasing the tree-level Higgs boson mass and ameliorating the need for large loop corrections.

6 Conclusion

It is well known that the scale invariant NMSSM has lower fine-tuning than the MSSM, but suffers from the domain wall problem. In this paper we have proposed a new version of the scale invariant NMSSM, called the NMSSM+, which introduces extra matter in order to reduce even more the fine-tuning of the NMSSM. This is not the first time that adding extra matter to the NMSSM to reduce fine-tuning has been considered, however usually the extra matter that is added is motivated by gauge mediated SUSY breaking [17]. In this paper the extra matter descends from an E_6 gauge group and fills out three complete 27-dimensional representations at the TeV scale, as in the E_6 SSM. However the $U(1)_N$ gauge group of the E_6 SSM is broken at a high energy scale leading to reduced fine-tuning relative to the fine-tuning in the E_6 SSM.

One of the motivations for introducing the NMSSM+ is that we have shown that the E_6 SSM as usually realised requires significant tree-level fine-tuning due to experimental limits on the mass of its Z'_N gauge boson. However, if the extra $U(1)_N$ gauge symmetry of the E_6 SSM is broken at a high energy scale by extra fields in an approximately D -flat direction, then this relaxes the fine-tuning considerably. From this point of view, the NMSSM+ may be regarded as the E_6 SSM with $U(1)_N$ gauge symmetry broken at a high energy scale, with associated scale invariant trilinear singlet couplings. This then leads to a low energy effective NMSSM+ that resembles the NMSSM with extra matter.

The resulting low energy NMSSM+ is summarised in equation (20), which approximates to equation (24) (or, equivalently, equation (14)). This low energy NMSSM+ represents a very complete formulation of the scale invariant NMSSM plus extra matter, including explicit couplings for the extra matter. Much of the low energy phenomenology of the extra matter has been discussed in the context of the E_6 SSM, however there are some important differences. For one thing, there is no low energy $U(1)_N$ gauge group or associated Z'_N gauge boson in the NMSSM+, since this gauge group is broken at a very high energy scale. Also, all three singlets S_i gain VEVs in the NMSSM+, with S_3 being responsible for the $\mu = \lambda \langle S_3 \rangle$ term, while $S_{1,2}$ are responsible for D and \bar{D} masses. This division of labour between the three singlets allows $S_{1,2}$ VEVs to be larger than the S_3 VEV, leading to the D and \bar{D} masses being larger than the $\mu = \lambda \langle S_3 \rangle$ term, while keeping λ quite large in order to maximise the tree-level contribution to the Higgs boson mass.

The high energy NMSSM+ in equation (16) provides a resolution of the domain wall problem of the NMSSM due to a discrete R -symmetry, which also stabilises the proton. The renormalisable part of the high energy model in equation (27) contains an explicit sector which breaks the $U(1)_N$ gauge symmetry, while the non-renormalisable terms lead to trilinear singlet couplings, and other higher order terms responsible for destabilising the cosmological domain walls. We gave options for discrete R -symmetries that can allow the scenario to be realised and which forbid rapid proton decay and avoid the domain wall

problems. We also explored approximate flavour symmetries that can suppress flavour changing neutral currents and naturally allow for a slight difference between the (radiatively induced) scales of EWSB and of exotic, coloured fermion masses.

Finally we recall there are two equivalent ways of looking at the NMSSM+, namely either as an extension of the scale invariant NMSSM with the exotic sector of the E_6 SSM, or as an extension of the E_6 SSM by the inclusion of cubic singlet terms (with the $U(1)_N$ broken near the GUT scale). Either way, the NMSSM+ has some remarkable features compared to the NMSSM or the E_6 SSM. In particular it solves the domain wall problem of the scale invariant NMSSM via a discrete R -symmetry and is expected to exhibit less overall fine-tuning than either the scale invariant NMSSM or the E_6 SSM.

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